Quantifying Resilience of Ocean Circulation in Simple Box Models

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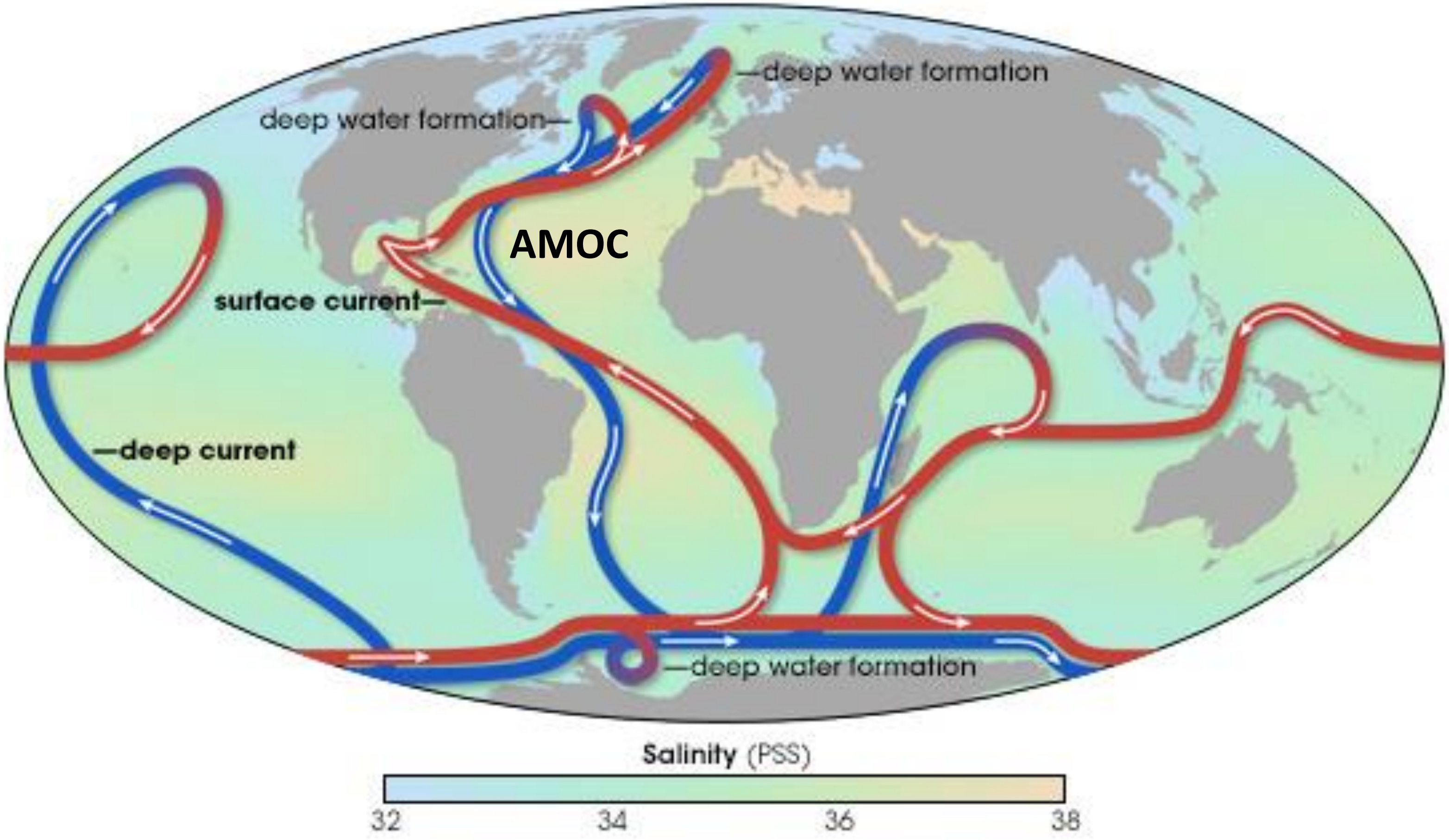
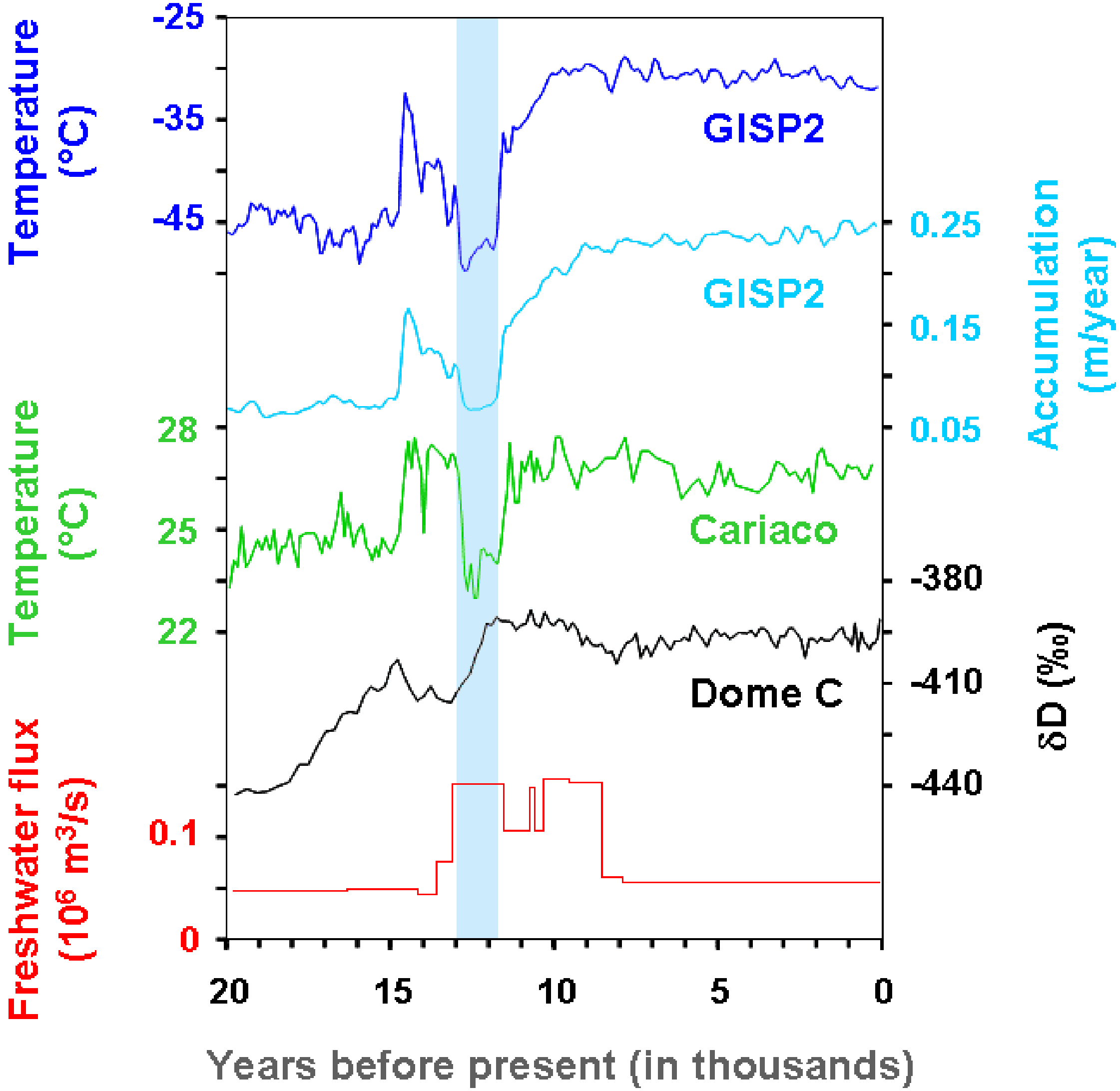
Mathematics of Climate Seminar
University of Minnesota
October 27, 2015

Oceans: a key player in climate dynamics



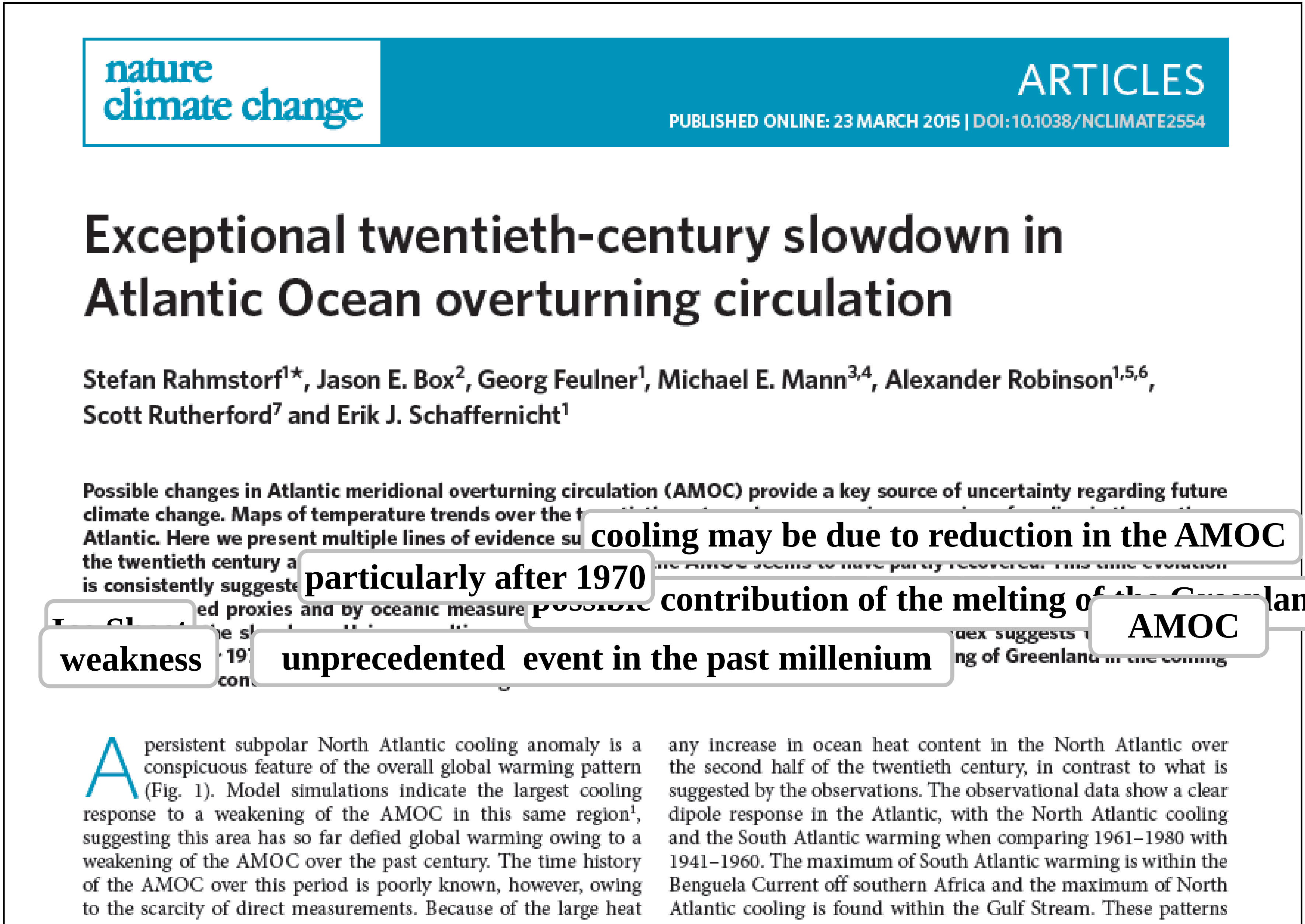
http://earthobservatory.nasa.gov/Features/Paleoclimatology_Evidence/paleoclimatology_evidence_2.php

Previous changes to circulation



http://www.ncdc.noaa.gov/paleo/abrupt/data4.html

Current changes to circulation



nature climate change

ARTICLES

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Exceptional twentieth-century slowdown in Atlantic Ocean overturning circulation

Stefan Rahmstorf[1*], Jason E. Box[2], Georg Feulner[1], Michael E. Mann[3,4], Alexander Robinson[1,5,6], Scott Rutherford[7] and Erik J. Schaffernicht[1]

Possible changes in Atlantic meridional overturning circulation (AMOC) provide a key source of uncertainty regarding future climate change. Maps of temperature trends over the t

Atlantic. Here we present multiple lines of evidence su

the twentieth century a

is consistently suggeste

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dex suggests

19

ng of Greenland

con

cooling may be due to reduction in the AMOC

particularly after 1970

possible contribution of the melting of the Greenland

Ice Sheet

AMOC

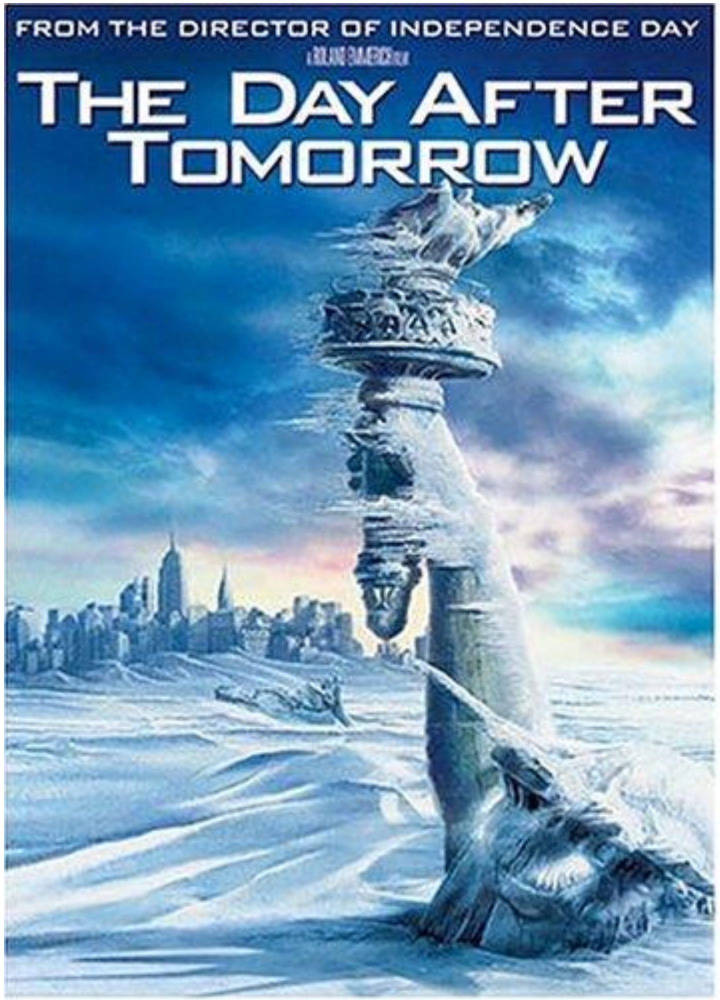
weakness

unprecedented event in the past millenium

A persistent subpolar North Atlantic cooling anomaly is a conspicuous feature of the overall global warming pattern (Fig. 1). Model simulations indicate the largest cooling response to a weakening of the AMOC in this same region[1], suggesting this area has so far defied global warming owing to a weakening of the AMOC over the past century. The time history of the AMOC over this period is poorly known, however, owing to the scarcity of direct measurements. Because of the large heat

any increase in ocean heat content in the North Atlantic over the second half of the twentieth century, in contrast to what is suggested by the observations. The observational data show a clear dipole response in the Atlantic, with the North Atlantic cooling and the South Atlantic warming when comparing 1961–1980 with 1941–1960. The maximum of South Atlantic warming is within the Benguela Current off southern Africa and the maximum of North Atlantic cooling is found within the Gulf Stream. These patterns

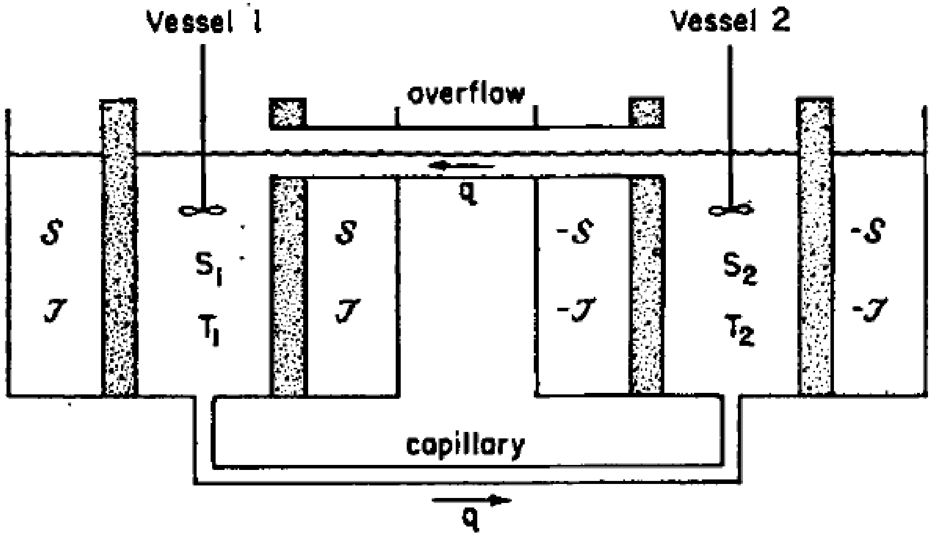
Science Fiction



http://ecx.images-amazon.com/images/I/51JSE1F1G9L.jpg

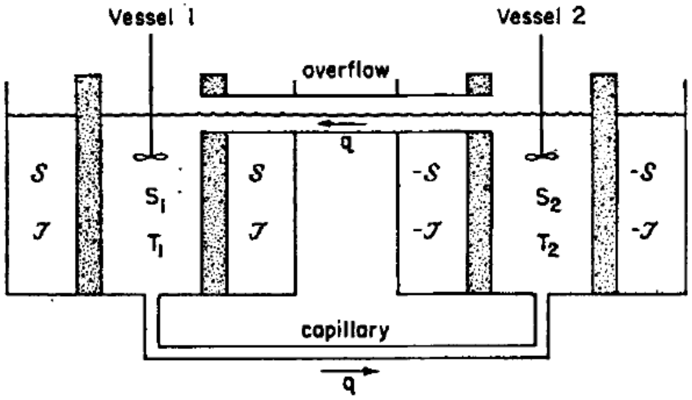
Q: How resilient is the AMOC to disruptions?

[**resilience:**
the capacity of a system to absorb disturbance
and maintain its basic structure and function]



(Stommel 1961)

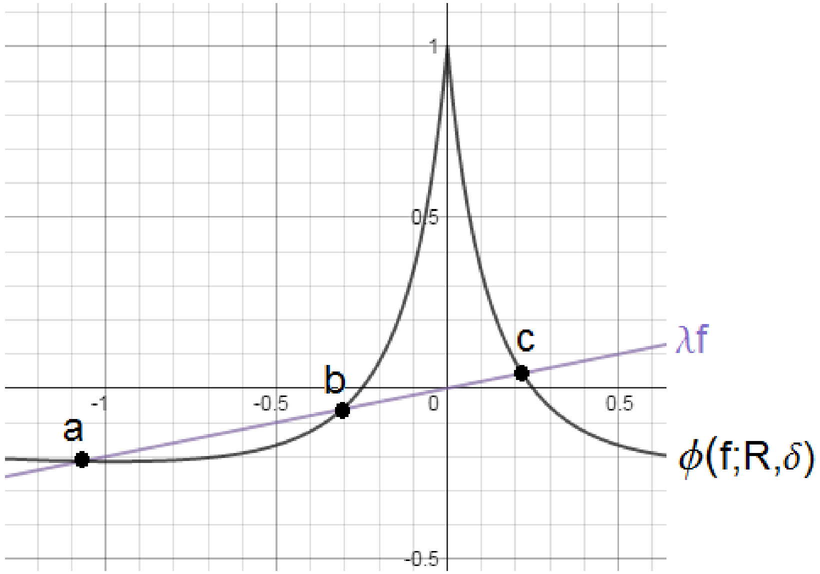
Stommel's Model

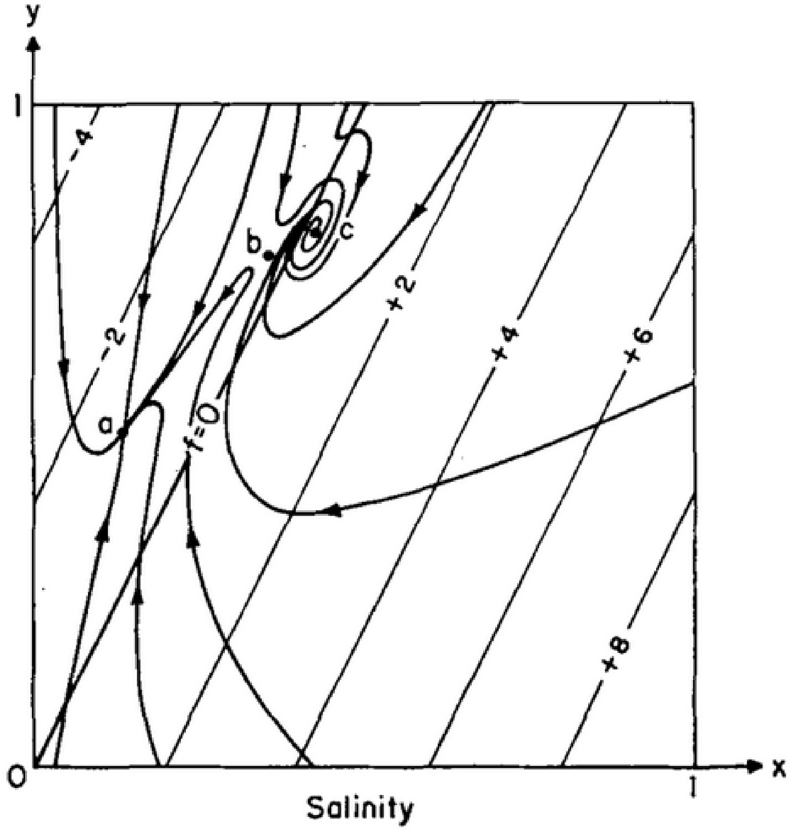


temp $y' = 1 - y - |f|y$

salinity $x' = \delta(1 - x) - |f|x$

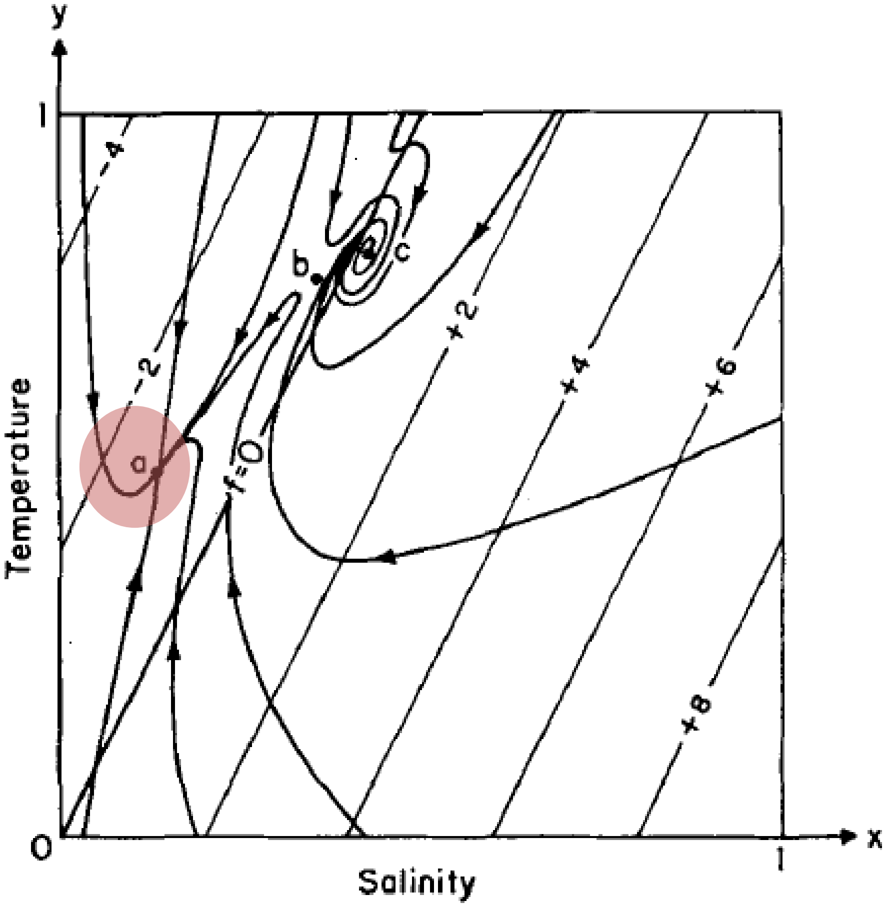
flow $\lambda f = (-y + Rx)$





[Stommel 1961]

How resilient is circulation equilibrium **a**?

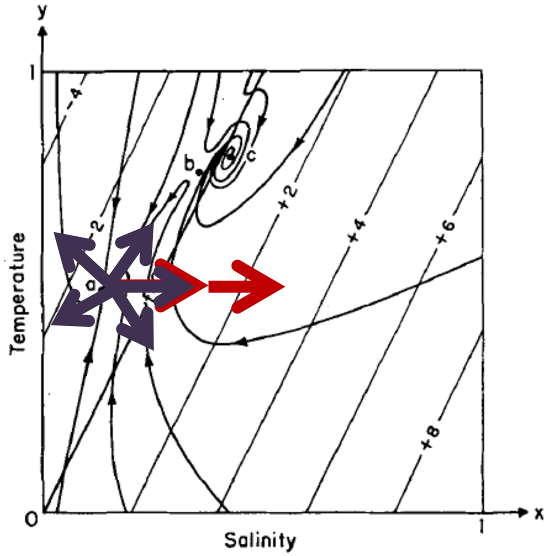


… the answer depends on disruptions considered!

Possible Disruptions

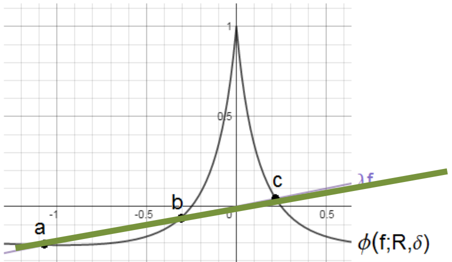
To State Variables:

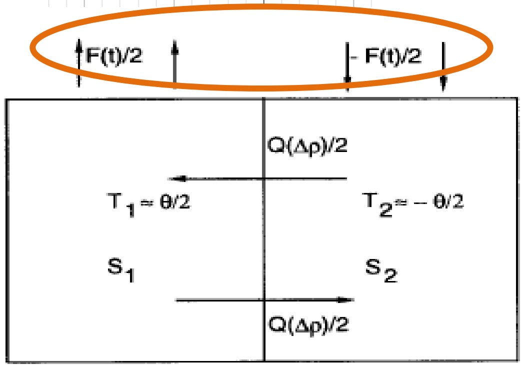
1. Repeated salinity “kicks”
2. Repeated “kicks” in any direction



To Parameters:

1. changes to parameter λ
2. changes to salinity forcing (Cessi’s adaptation)





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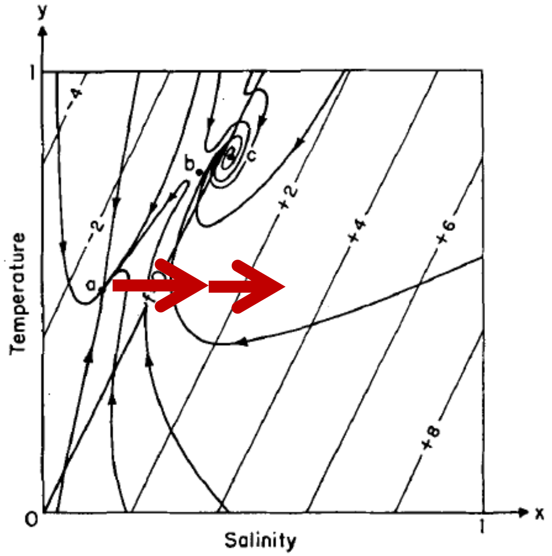

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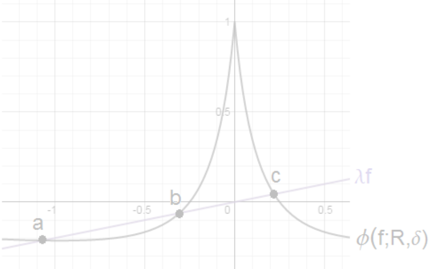
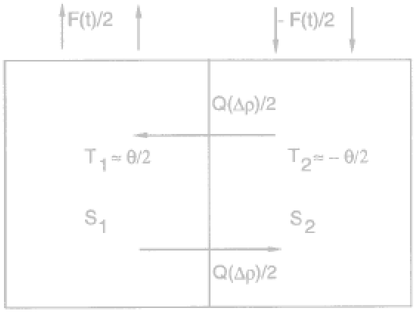
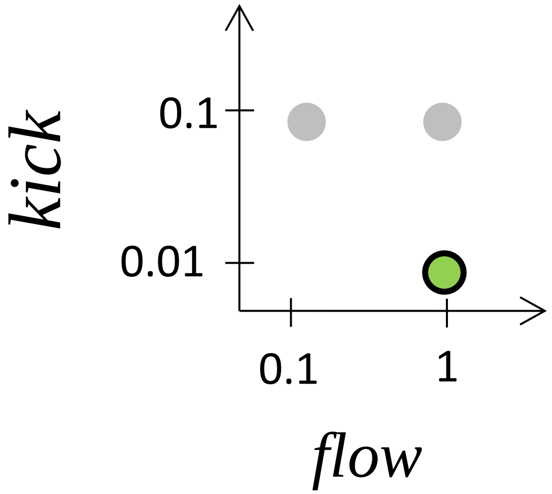
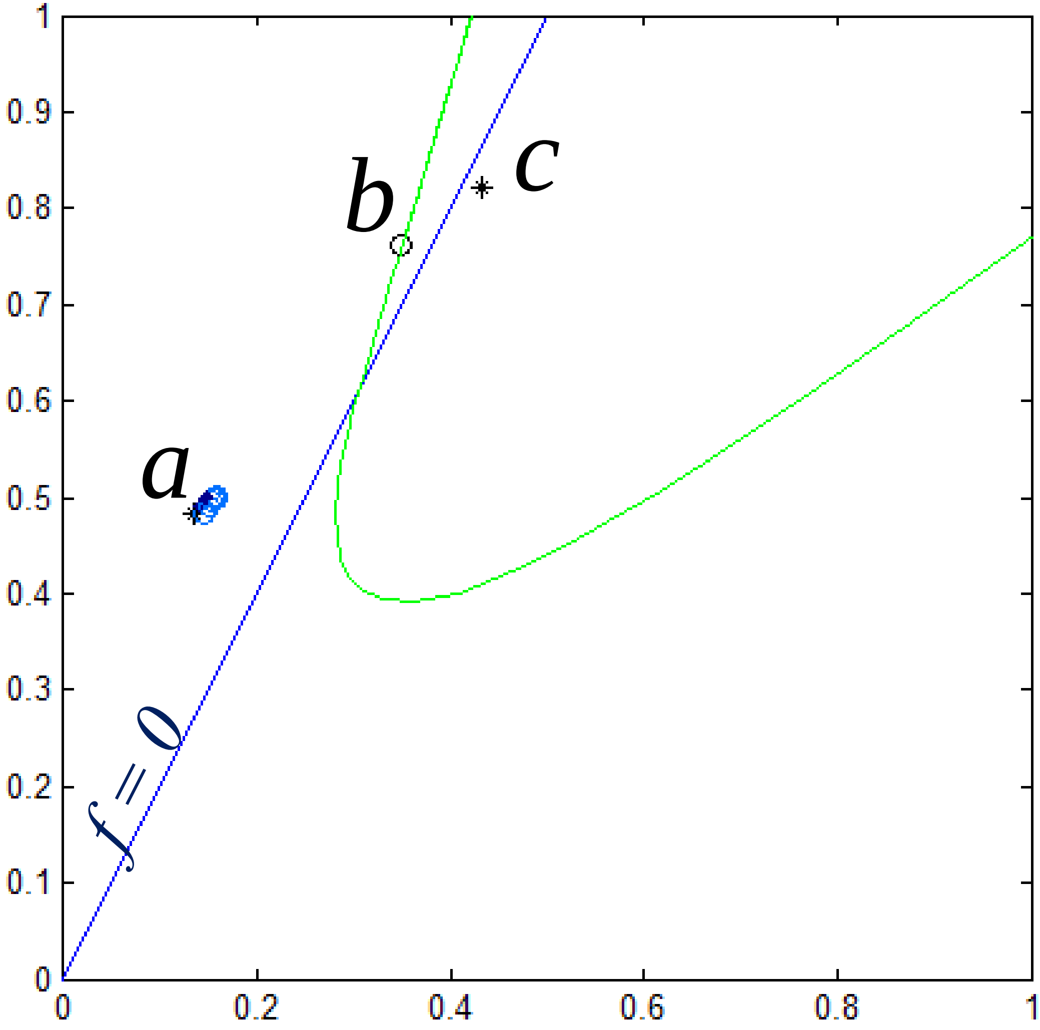


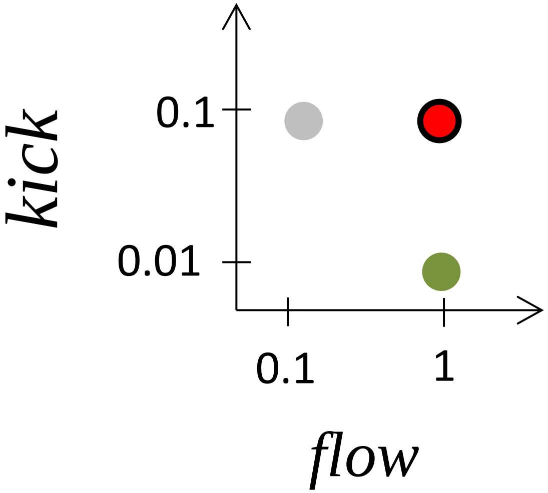


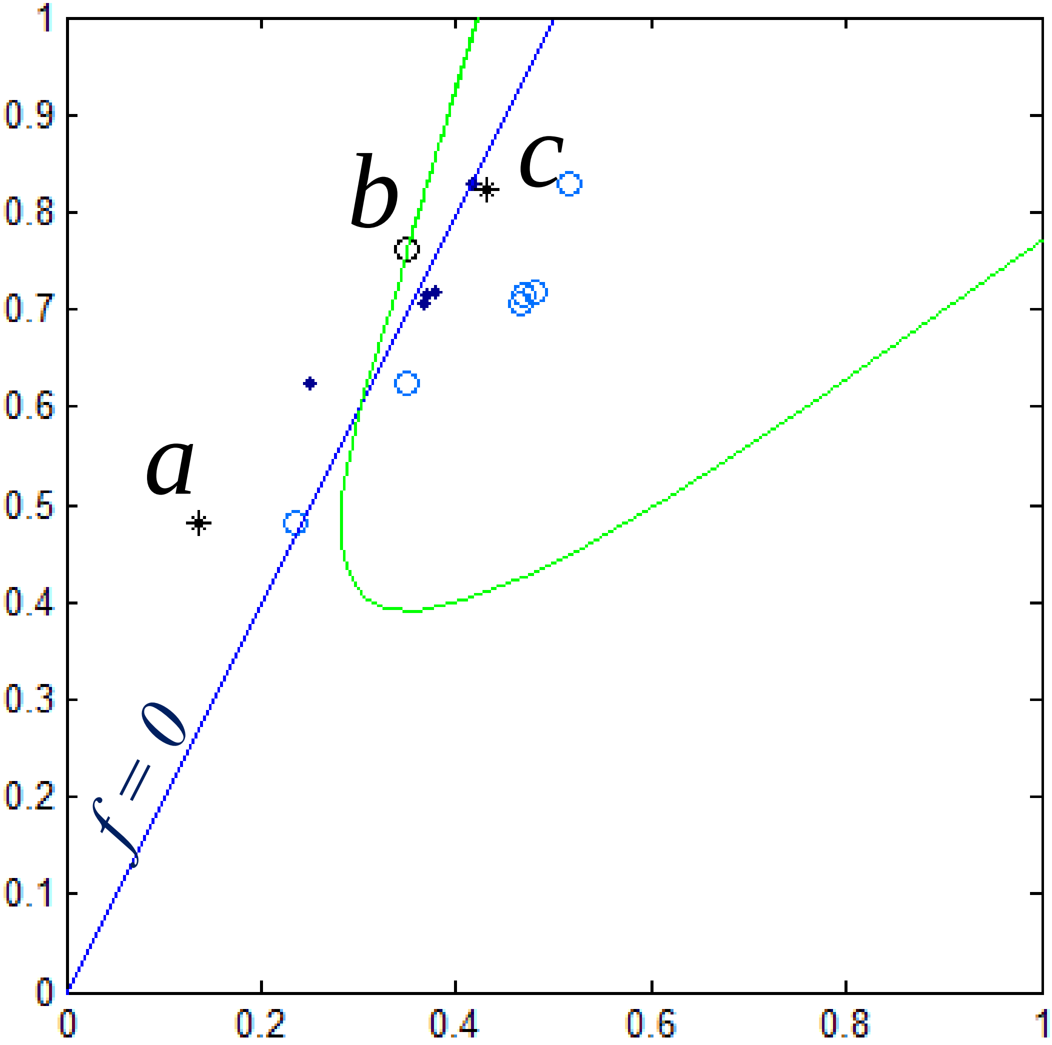
Small Kicks, Long Recoveries



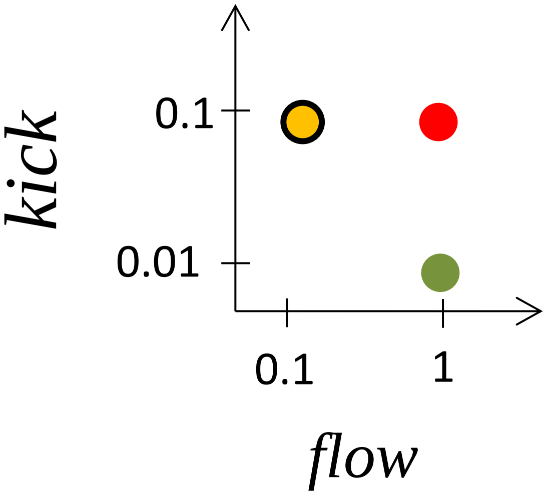


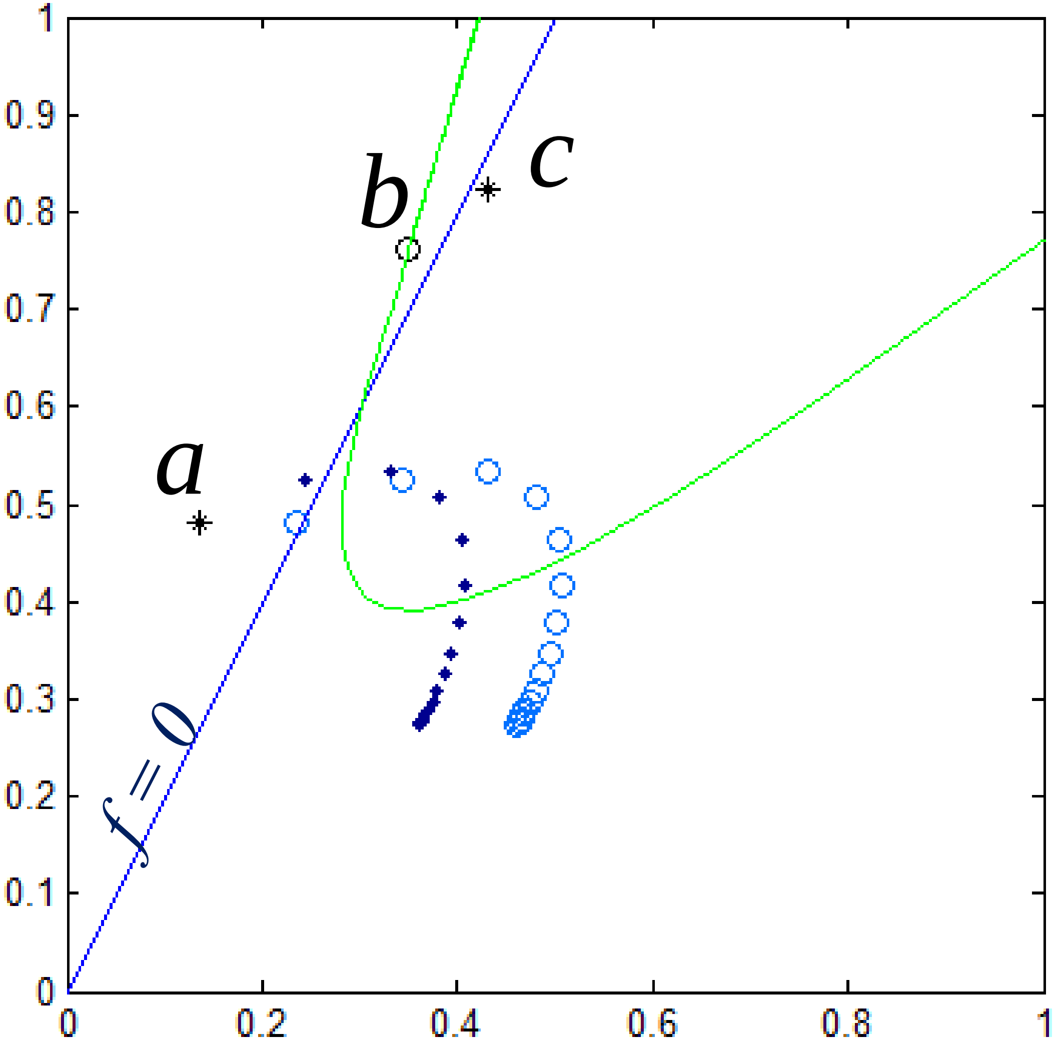
Big Kicks, Long Recoveries





Big Kicks, Short Recoveries





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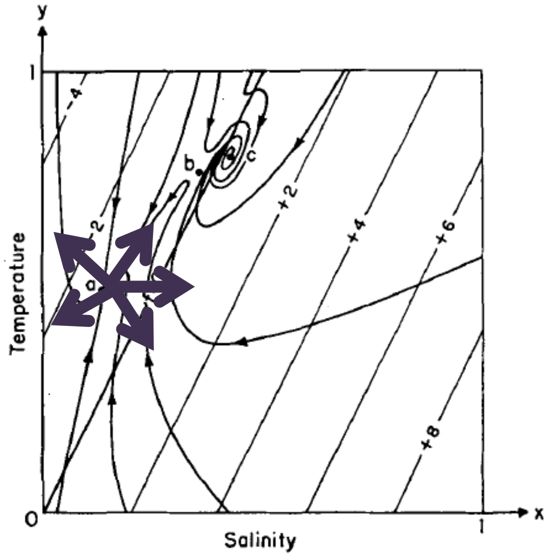

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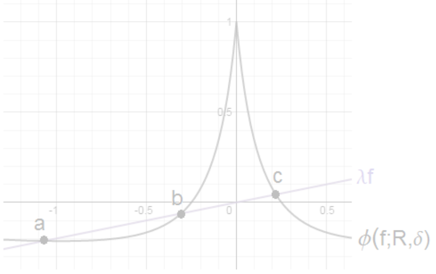
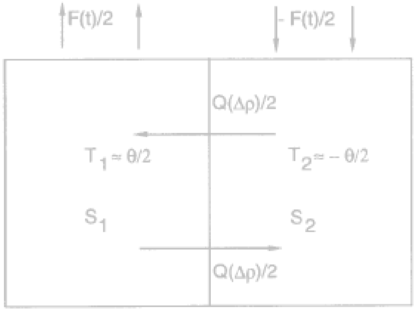
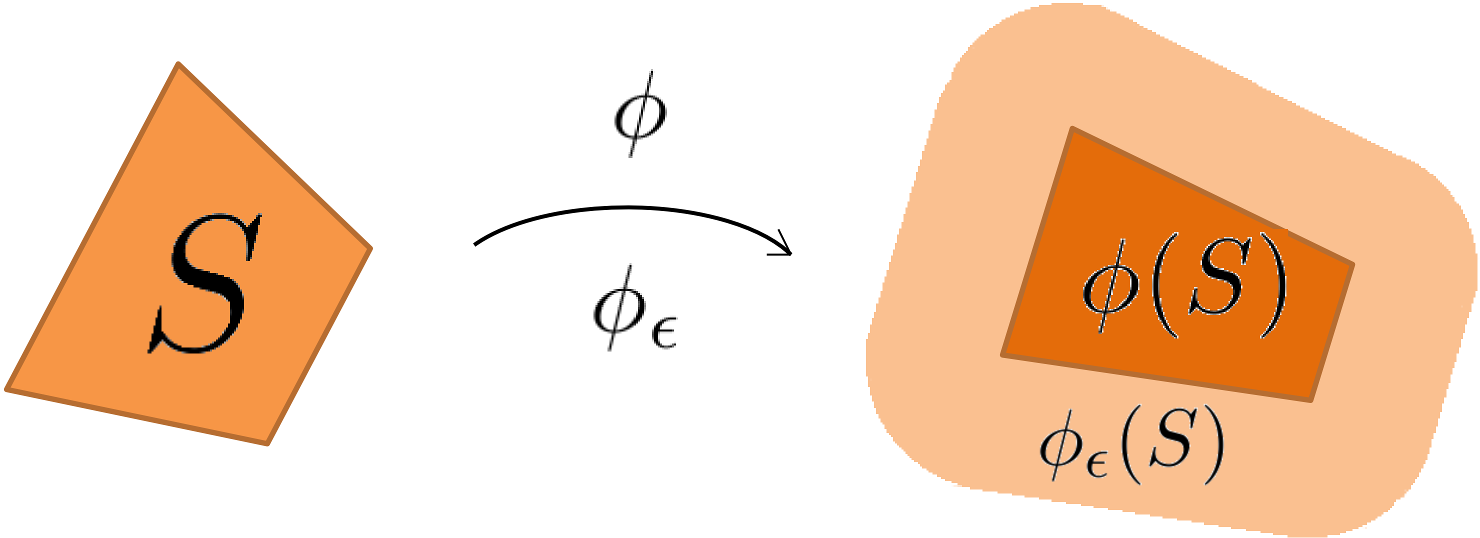




Resilience as “Intensity of Attraction”

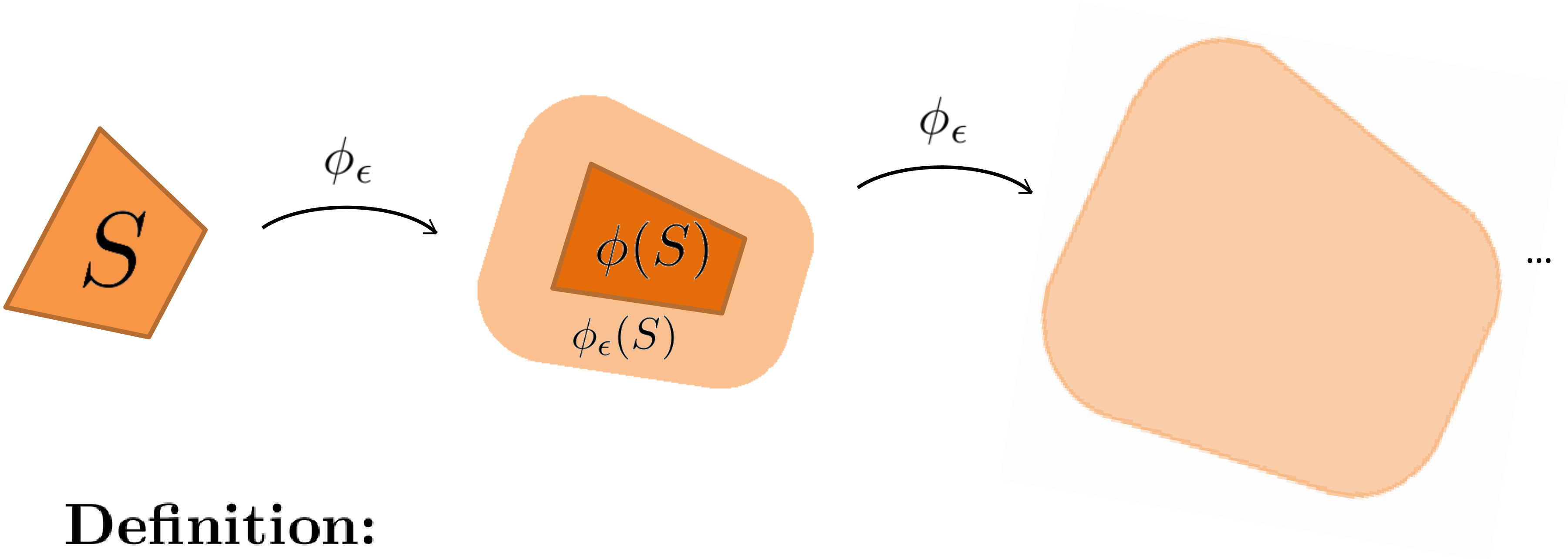
Define $\phi_\epsilon : \mathscr{P}(X) \to \mathscr{P}(X)$ by

$$\phi_\epsilon(S) = \{x \in X \mid \mathrm{dist}(x, \phi(S)) < \epsilon\}$$



(McGehee 1988)

Let $P_\epsilon(S)$ denote the set of all points accessible by ϵ-pseudo-orbits starting on S



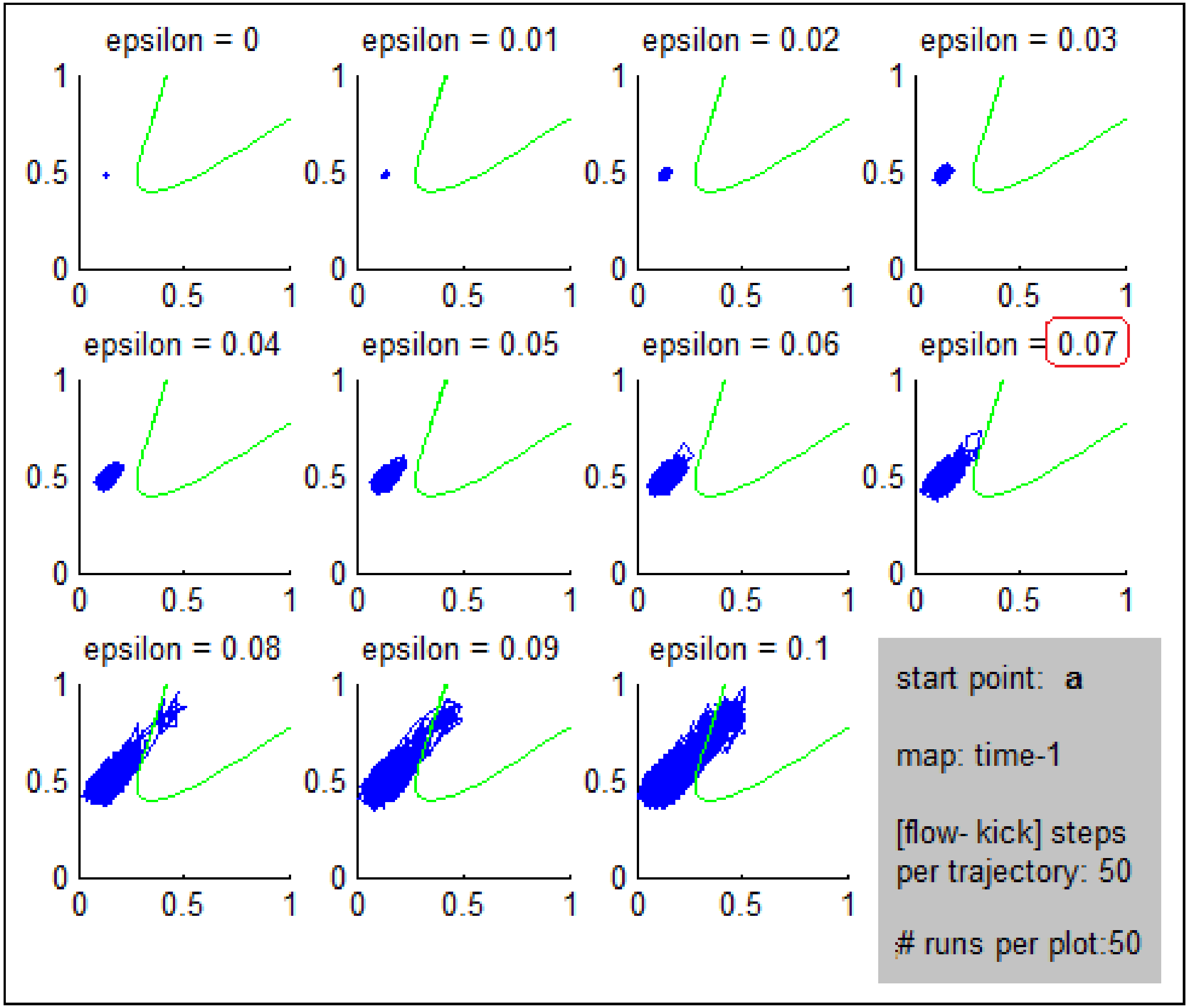
Definition:

The **chain intensity** of A is

$$\mu(A) \equiv \sup\{\epsilon \mid P_\epsilon(A) \subset \text{ compact set } \subset \mathcal{D}(A)\}$$

(McGehee 1988)

Chain Intensity of a



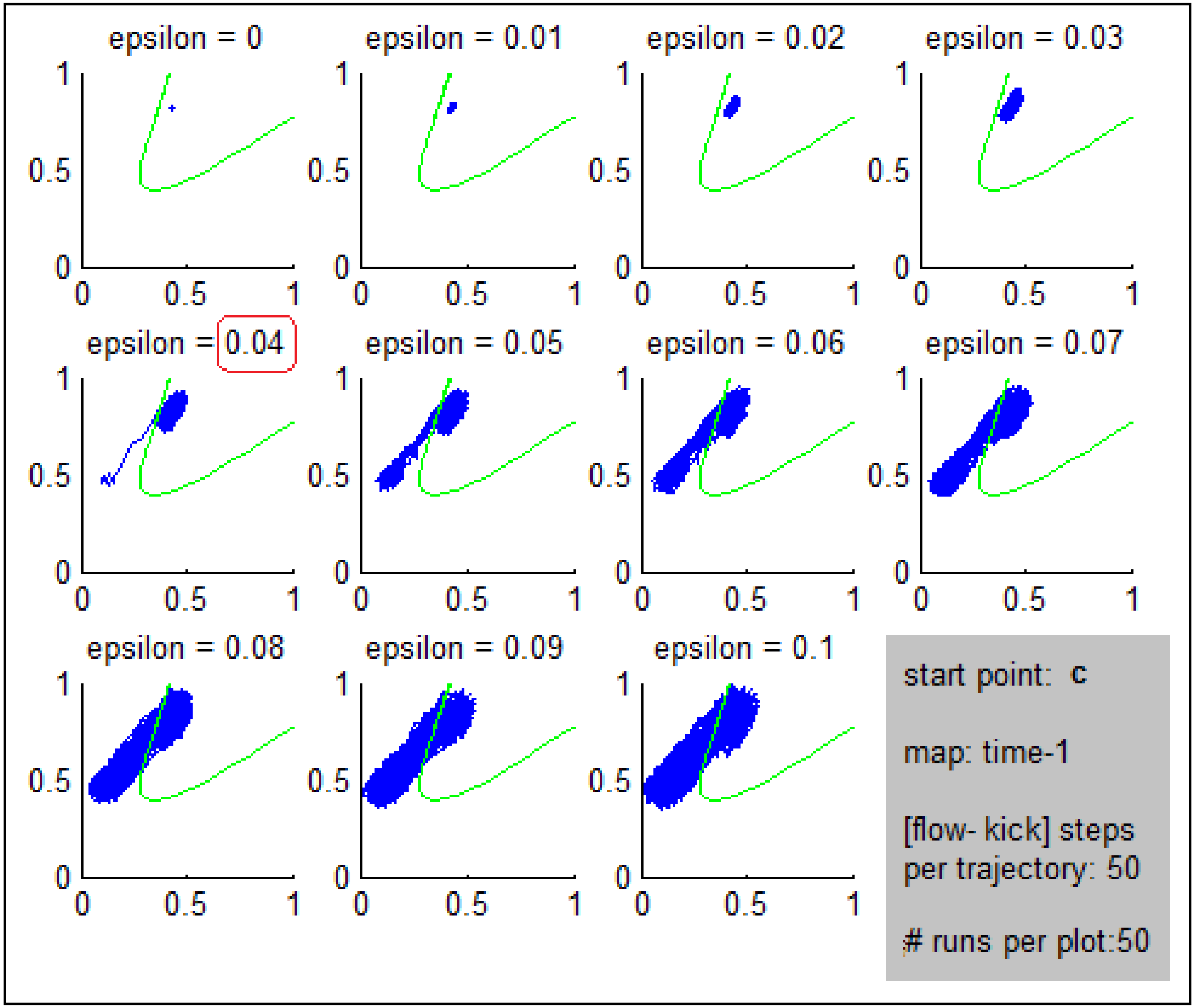
start point: **a**

map: time-1

[flow- kick] steps
per trajectory: 50

runs per plot:50

Chain intensity of $\boldsymbol{c}$



start point: **c**

map: time-1

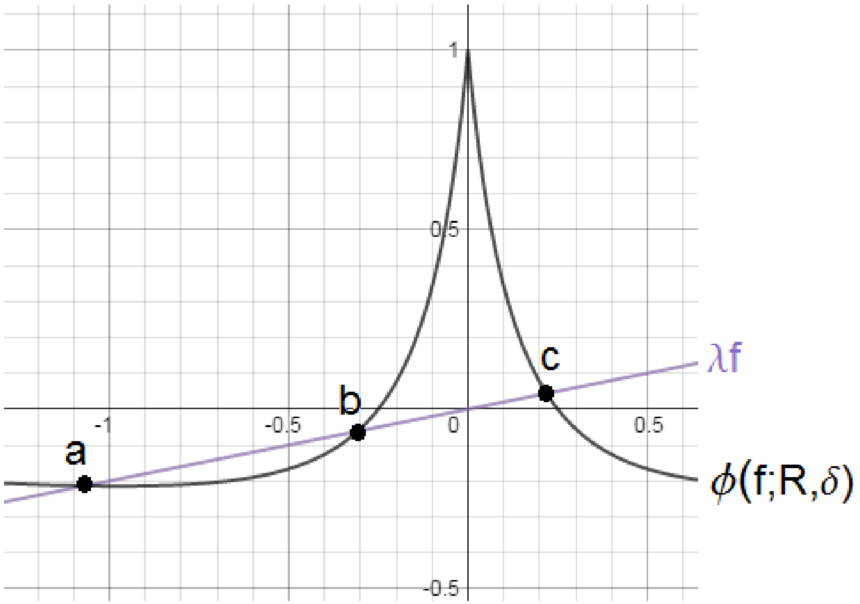
[flow- kick] steps per trajectory: 50

runs per plot:50

Conclusion: When $\lambda = 0.2$,

$$\mu(\mathbf{a}) \approx 0.7 \quad \text{and} \quad \mu(\mathbf{c}) \approx 0.4.$$

Is **a** *more resilient than* **c***?*



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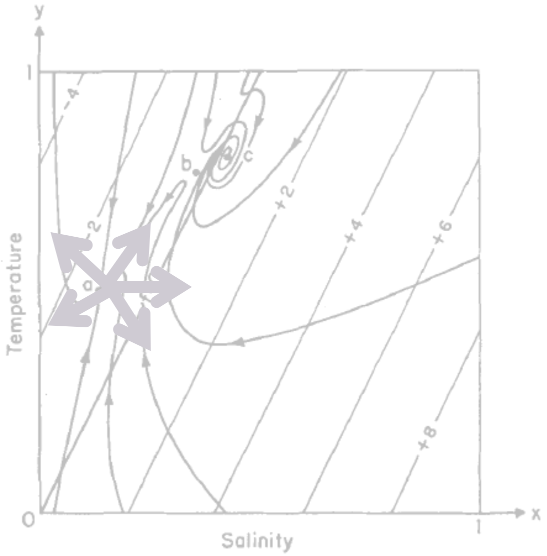

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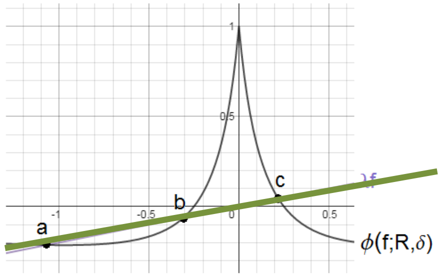
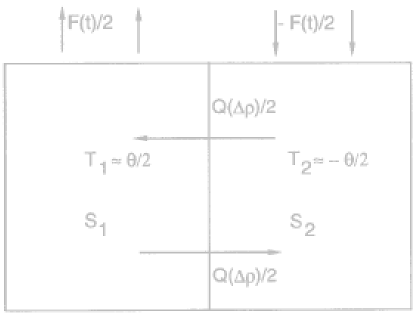
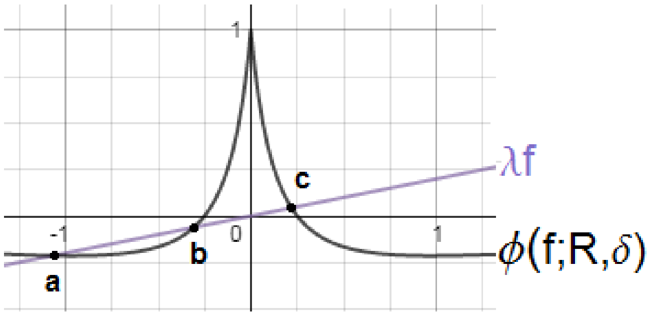
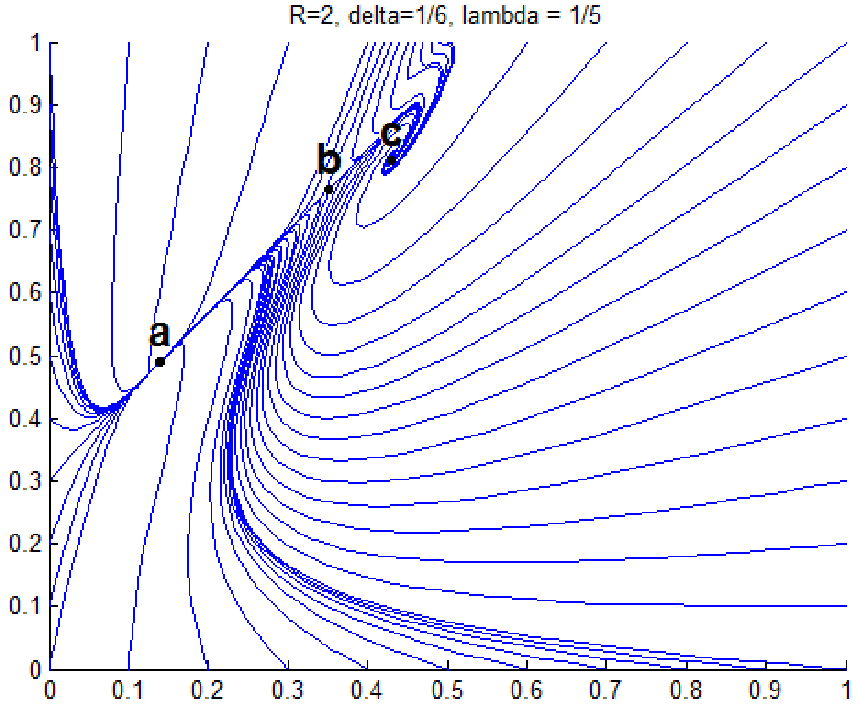




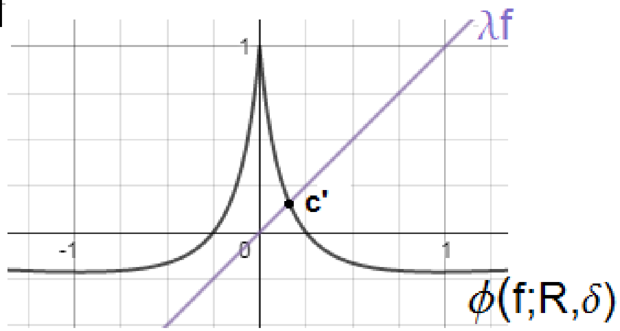
Alternate λ Values

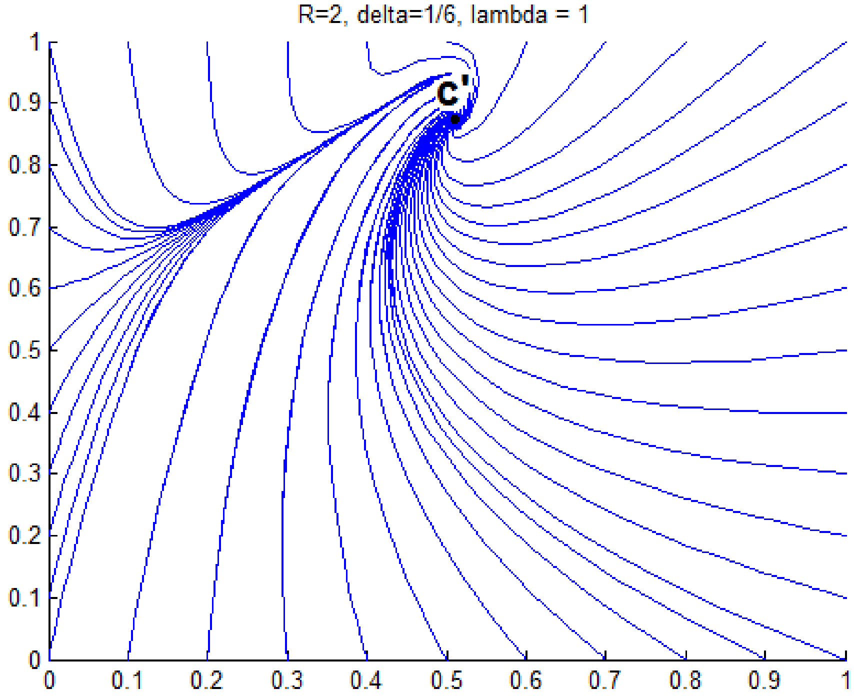
$\lambda = 1/5$



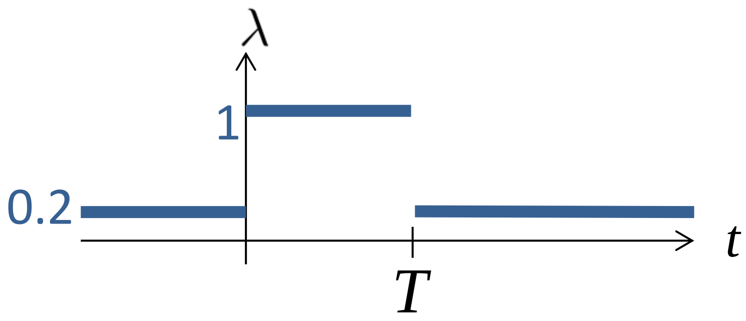

R=2, delta=1/6, lambda = 1/5

$\lambda = 1$

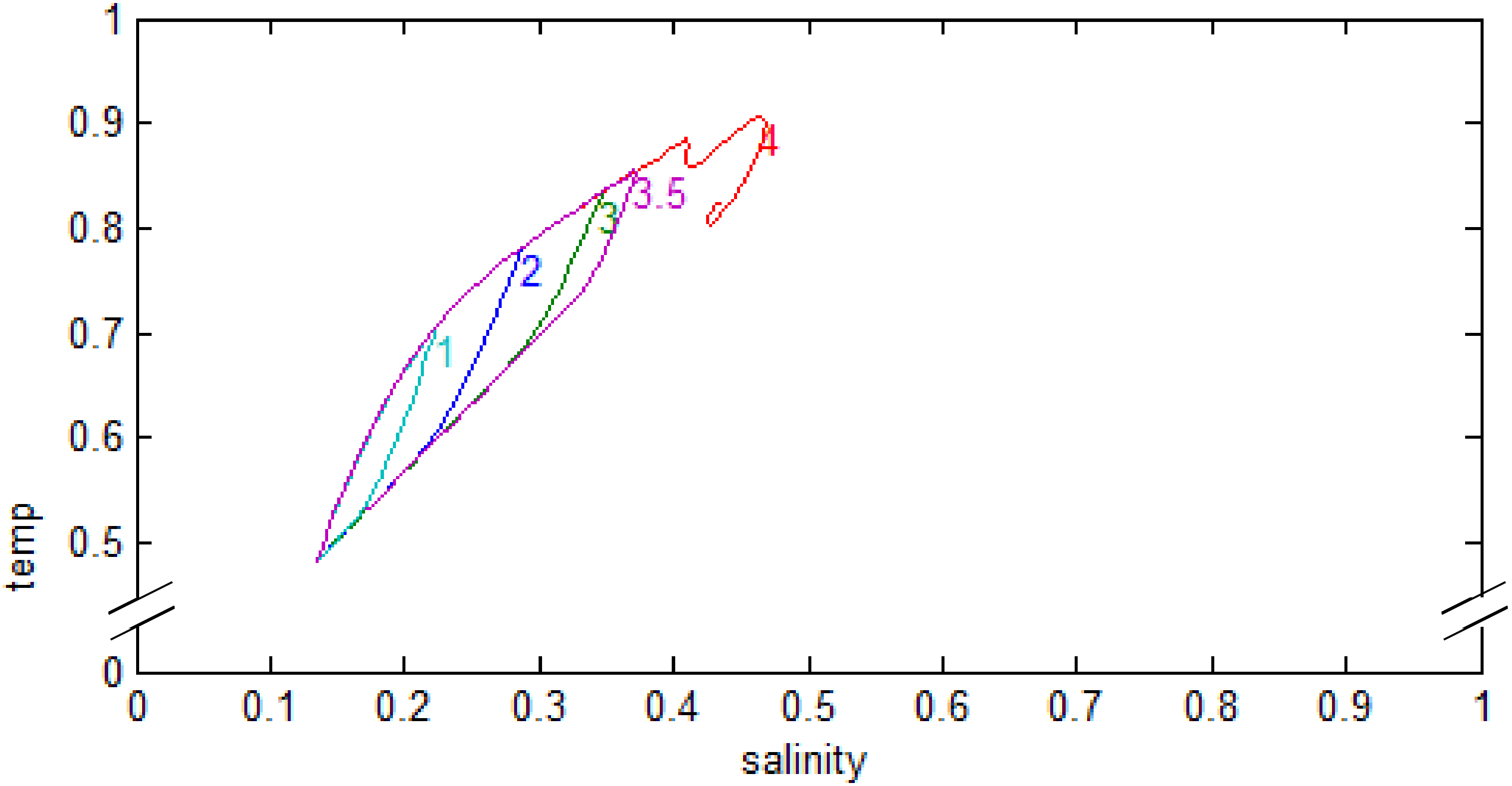



R=2, delta=1/6, lambda = 1

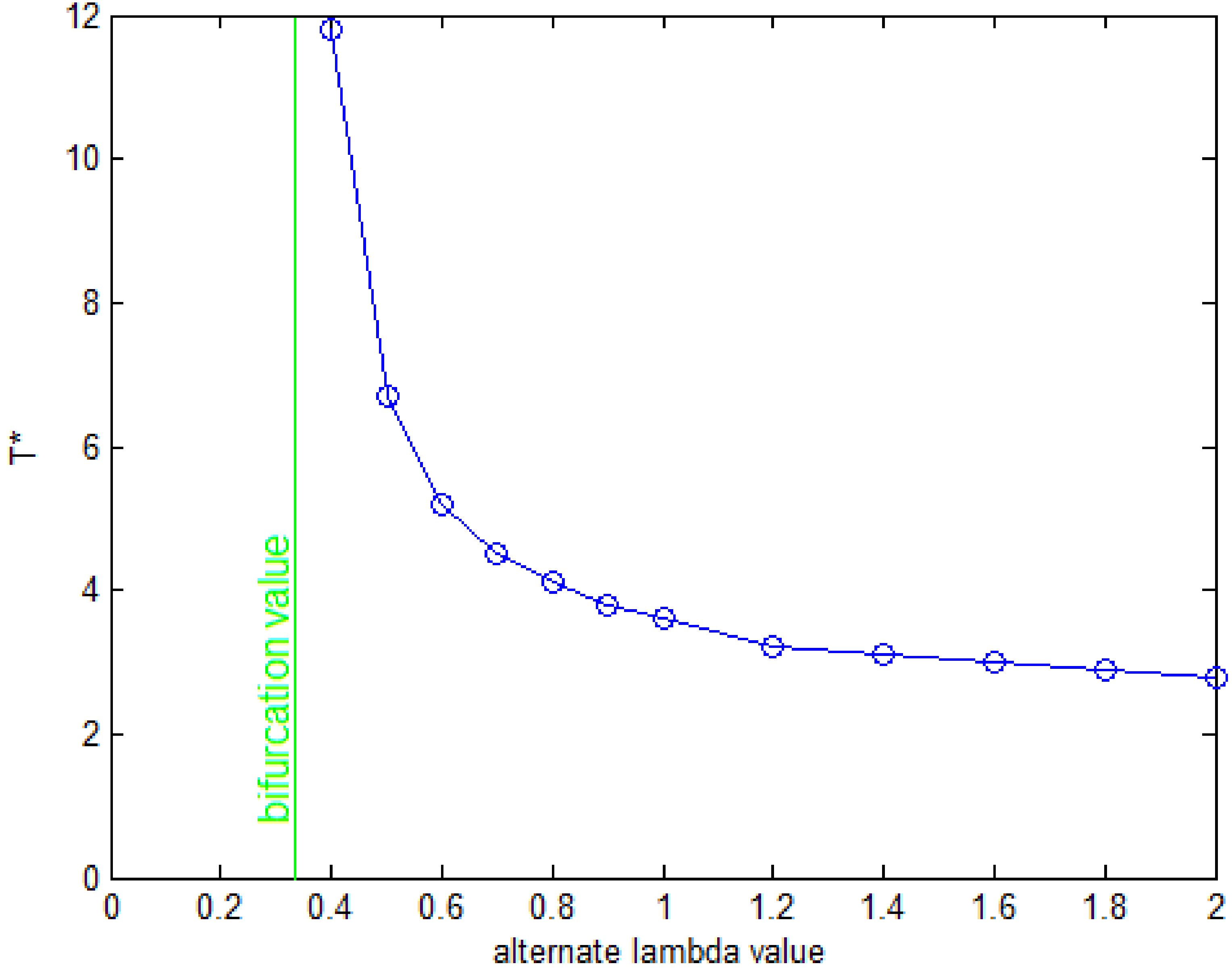
Parameter perturbation schedule:



Trajectories for different values of T:



Critical time as a function of alternate λ value



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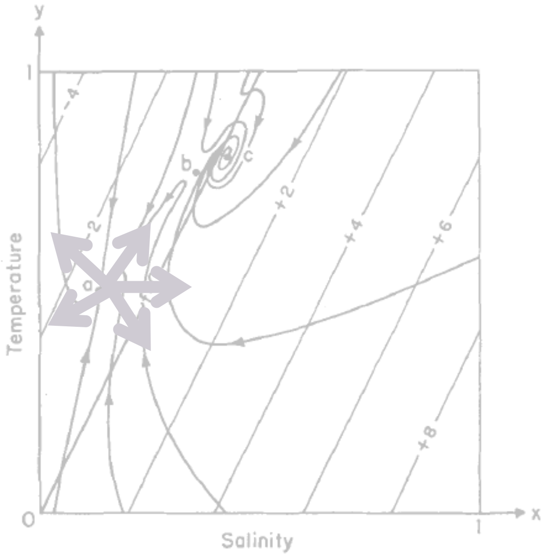

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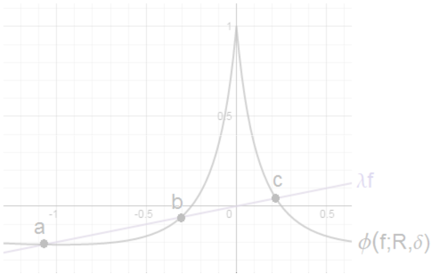

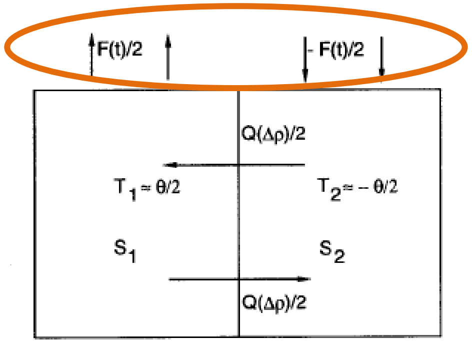

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